ACTIVITY CODE: 1903122021

B.Sc. 6th Semester (Honours) Examination, October 2020

Subject: Mathematics

Course ID: 62116 **Course Code: SH/MTH/603/DSE-3**

Course Title: Number Theory

Full Marks: 20 **Time: 1 Hour**

<u>The figures in the margin indicate full marks</u>

Unless otherwise mentioned the symbols have their usual meaning.

1. **Answer any *two* of the following questions:** 2×2=4

a) Find the solution of $155x + 45y = 7$.

b) Define prime counting function $\pi(x)$ and find $\pi(100)$.

c) If p is prime, then prove that $a^p \equiv a \ (mod\ p)$ for all a.

d) If a is prime to b and c then prove that a is prime to bc.

e) Solve the linear congruence $5x \equiv 3(\text{mod } 11)$.

f) Evaluate the Legendre symbols $\left({}^{71}\!/_{73}\right)$ and $\left({}^{196}\!/_{23}\right)$.

g) Verify that 2 is a primitive root of 19, but not of 17.

h) For $n > 2$, prove that $\phi(n)$ is an even integer.

2. **Answer *any two* of the following questions:** 5×2=10

a) (i) Prove that a Diophantine equation $ax + by = c$ has an integral solution if and only if $\gcd(a,b) | c$.

(ii) Find the remainder when $1! + 2! + \ldots\ldots\ldots\ldots + 99! + 100!$ is divided by 12. 3+2

b) (i) If f is a multiplicative function and F is defined by

$$F(n) = \sum_{d|n} f(d)$$

then show that F is also multiplicative.

(ii) Find the last two digits of 3^{256}. 3+2

c) (i) Prove that the area of a Pythagorean triangle can never be equal to a perfect square.

(ii) Encrypt the massage 'RETURN HOME' using Caesar cipher. 3+2

d) Solve the linear congruence $17x \equiv 9 \ (mod\ 276)$.

e) Prove that

(i) $\phi(3n) = 3\phi(n)$ if and only if $3|n$,

(ii) $if\ \phi(n) = \frac{n}{2}$ then $n = 2^k$ for some $k \geq 1$. 3+2

f) Prove that the integer 2^k has no primitive root for $k \geq 3$.

3. Answer *any two* from either a) or b) 2×3=6

a) (i) Find the remainder when $2(26!)$ is divided by 29.

(ii) Prove that for $n > 1$, the sum of the positive integers less than n and relatively prime to n is $\frac{1}{2}n\phi(n)$.

(iii) Prove that there are infinitely many primes of the form $4k + 1$.

(iv) Solve the system of linear congruences

$$x \equiv 2(\text{mod } 3)$$
$$x \equiv 3(\text{mod } 5)$$
$$x \equiv 2(\text{mod } 7).$$

b) (i) If n is a positive integer and 'a' is prime to n then prove that $a^{\phi(n)} \equiv 1(\text{mod } n)$.

(ii) Use the Hill cipher

$$C_1 \equiv 5P_1 + 2P_2 \ (\text{mod } 26)$$
$$C_2 \equiv 3P_1 + 4P_2 \ (\text{mod } 26)$$

to encipher the massage 'GIVE THEM TIME'.

(iii) Solve the following quadratic congruence

$$x^2 + 7x + 10 \equiv 0(\text{mod } 11).$$

(iv) Prove that $ax \equiv ay(\text{mod } m)$ if and only if $x \equiv y\left(\text{mod } \frac{m}{d}\right)$ where $d = \gcd(a, m)$.

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ACTIVITY CODE: 1903123021

B.Sc. 6th Semester (Honours) Examination, October 2020

Subject: Mathematics

Course ID: 62116 **Course Code: SH/MTH/603/DSE-3**

Course Title: Mechanics

Full Marks: 20 **Time: 1 Hour**

<u>The figures in the margin indicate full marks</u>

Unless otherwise mentioned the symbols have their usual meaning.

1. **Answer *any two* of the following questions:** (2×2=4)

a) When is a statical equilibrium said to be unstable?

b) Define wrench and pitch.

c) State principle of virtual work.

d) Define limiting or terminal velocity of a particle moving in a vertical line downwards in a resisting medium.

e) A particle moves along a straight line according to the law $s^2 = 6t^2 + 4t + 3$ where s is the distance at time t. Prove that the acceleration varies as $\frac{1}{s^3}$.

f) State D' Alembert's principle.

g) Obtain the product of inertia for a uniform elliptic lamina with respect to the major axis.

h) Prove or disprove: The coefficient of friction is equal to the cotangent of the angle of friction.

2. **Answer *any two* of the following questions:** (5×2=10)

a) Forces X, Y, Z act along the three straight lines

$$y = b, z = -c;\ z = c, x = -a;\ x = a, y = -b$$

respectively. Show that they will have a single resultant if $\frac{a}{X} + \frac{b}{Y} + \frac{c}{Z}$=0.

b) A string of length a forms the shorter diagonal of a rhombus formed of four uniform rods, each of length b and weight W, which are hinged together. If one of the rods be suspended in a horizontal position, prove that the tension of the string is

$$\frac{2W(2b^2 - a^2)}{b\sqrt{4b^2 - a^2}}.$$

c) A particle moves with a central acceleration μr^{-2}. It is projected with a velocity V at a distance R. Show that the path is a rectangular hyperbola, if the angle of projection θ is given by

$$\sin\theta = \mu \div [VR\left(V^2 - \frac{2\mu}{R}\right)^{\frac{1}{2}}].$$

d) A car of mass m starts from rest and moves on a level road under a constant frictional resistance, the engine working at a constant rate P. If the maximum speed be $V$and the speed u be attained after travelling a distance s in time t, then show that

$$t = \frac{s}{V} + \frac{mu^2}{2P}.$$

e) A weightless rod AOB can turn freely in a vertical plane about a smooth fixed hinge at O. Two heavy particles of masses M and M' are attached to the rod at A and B and oscillate with it. If $OA = a$ and $OB = b$ and θ be the angle that the rod makes with the vertical at time t, then show that the equation determining the motion is given by

$$(Ma^2 + M'b^2)\ddot{\theta} + (Ma + M'b)g\sin\theta = 0$$

f) A rod of length $2a$ is suspended by a string of length l, attached to one end; if the string and rod revolve about the vertical with a uniform angular velocity, and their inclination to the vertical be θ and φ respectively, then show that

$$\frac{3l}{a} = \frac{(4\tan\theta - 3\tan\varphi)\sin\varphi}{(\tan\varphi - \tan\theta)\sin\theta}$$

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3. Answer *any two* from either a) or b) (2×3=6)

a) (i) Find the centre of gravity of a semi-circular plate of radius a whose mass per unit area at any point varies as $\sqrt{a^2 - r^2}$, where r is the distance of the point from the centre.

(ii) Show that the equation of the momental ellipsoid at the centre of an elliptic plate is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \left(\frac{1}{a^2} + \frac{1}{b^2}\right)z^2 =$ constant.

(iii) A particle is projected in a resisting medium whose resistance is k×(velocity) and the initial velocity of the particle is *V*. Show that the velocity of the particle in time *t* is $v = Ve^{-k}$.

(iv) A rhombus *ABCD* is formed of four equal uniform rods freely jointed together and suspended from the point *A*; it is kept in position by a light rod joining the mid-points of *BC* and *CD*. If *T* be the thrust in this rod and *W* the weight of the rhombus, prove by the principle of virtual work that $T = Wtan\frac{\alpha}{2}$, where α denotes the angle at the point *A*.

b) (i) *AB* and *BC* are two equal similar rods freely hinged at *B* and lie in a straight line on a smooth table. The end *A* is struck by a blow perpendicular to *AB*. Show that applied impulse at *A* is four times that of the impulse at *B*.

(ii) A solid cone of semi-vertical angle α and height *h* swings as a compound pendulum about a horizontal diameter of its base. Show that the length of the equivalent simple pendulum is $\frac{h}{5}(2 + 3tan^2\alpha)$.

(iii) Three forces each equal to *P* act on a body, one at the point (*a*,0,0) parallel to *y*-axis, the second at the point (0,*b*,0) parallel to *z*-axis and the third at the point (0,0,*c*) parallel to *x*-axis, the axes being rectangular. Find magnitude of the resultant wrench.

(iv) If the nearly circular orbit of a particle be $p^2(a^{m-2} - r^{m-2}) = b^m$, show that the apsidal angle is nearly $\frac{\pi}{\sqrt{m}}$.

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