ACTIVITY CODE: 1903127021

B.Sc. 6th Semester (Programme) Examination, October 2020

Subject: Mathematics

Course ID: 62118 **Course Code: SP/MTH/601/DSE-1B**

Course Title: Mechanics

Full Marks: 20 **Time: 1 Hour**

The figures in the margin indicate full marks

Unless otherwise mentioned the symbols have their usual meaning.

1. Answer *any two* of the following questions: 2×2=4

a) Let a system of coplanar forces reduced to a single force *R* acting at a point *O* along with a couple *G*. Find the equation of the line of action of *R*.

b) Prove or disprove: The centre of gravity of a body is unique.

c) Find the moment of inertia of circular wire about a diameter of the wire.

d) To define the momental ellipsoid about a point *O* of a mass distribution, it is assumed that there is no line through *O* about which the moment of inertia vanishes. Explain the reasons.

e) Show that the angular momentum of a body about it axis of rotation is $Mk^2\dot{\theta}$.

f) A particle moving in a straight line is subject to a resistance which produces the retardation kv^2 where *v* is the velocity and *k* is a constant. Find *v* in terms of the distance *x* described by the particle. Given the initial velocity of the particle being *u*.

g) Write down two invariables for a system of forces acting on a rigid body.

h) What are the axis and vertical angle of the cone of friction.

2. Answer *any two* of the following questions: 5×2=10

a) A wire in the form of a semi-circle of radius *a*. Show that at an end of its diameter the principal axes in its plane are inclined to the diameter at angles $\frac{1}{2}tan^{-1}\frac{4}{\pi}$ and $\frac{\pi}{2}+\frac{1}{2}tan^{-1}\frac{4}{\pi}$.

b) A uniform rod of length $2a$ can turn freely about an end O which is fixed. It is started from the position in which it hangs vertically with such an angular velocity as can just take it to the highest position. Prove that the time of describing any angle $\theta(<\pi)$ is $\sqrt{\frac{4a}{3g}}\log\tan(\frac{\pi}{4}+\frac{\theta}{4})$.

c) A solid hemisphere rests on a plane inclined to the horizon at an angle $\alpha < sin^{-1}\frac{3}{8}$ and the plane is rough enough to prevent any sliding. Find the position of equilibrium and show that it is stable.

d) A force parallel to the axis of z acts at the point $(a,0,0)$ and an equal force perpendicular to the axis of z acts at the point $(-a,0,0)$. Show that the central axis of the system lies on the surface $z^2\,(x^2+y^2)=(x^2+y^2-ax)^2$.

e) A particle is projected in a medium whose resistance is proportional to the cube of the velocity and no other forces act on the particle. While the velocity diminishes from V_1 to V_2, the particle traverses a distance d in time t. Show that $\frac{d}{t}=\frac{2V_1V_2}{V_1+V_2}$.

f) Find the velocity and distance from the Earth's surface of a synchronous satellite, given the radius of the Earth=6400 km, acceleration due to gravity on Earth's surface=9.8m/sec^2. A synchronous satellite moves in the equatorial plane in a circular orbit with the same period of the Earth.

3. Answer *any two* from either a) or b): 3×2=6

a) (i) Forces P,Q, R act along three non-intersecting edges of a cube, find the central axis.

(ii) A uniform rod of mass M and length $2a$ is free to turn about one end O. A particle of mass $\frac{4M}{3}$ can be attached to the rod at any point. The square of the radius of gyration of the system about O is $\frac{4(a^2+x^2)}{7}$. Prove that the length of equivalent simple pendulum is least when the particle is at a distance $\frac{a}{2}$ from O.

(iii) The position of a particle of mass m moving in space referred to a set of rectangular axes at any instant t is $(a\cos(nt)\,, a\sin(nt), \frac{1}{2}at^2)$. Find the components and magnitudes of velocity and acceleration of the particle.

(iv) A solid frustum of paraboloid of revolution of height h and latus rectum $4a$ rests with its vertex on the vertex of paraboloid of revolution whose latus rectum is $4b$. Show that equilibrium is stable if $h < \frac{3ab}{a+b}$.

b) (i) The couple components of a system of coplanar forces when reduced with respect to two different bases A and B are G and H respectively. Show that the couple component when the system is reduced with respect to the middle point of AB is $\frac{1}{2}(G + H)$.

(ii) The lengths AB and AD of two sides of a rectangle $ABCD$ are $2a$ and $2b$ ($a>b$) respectively. Show that the inclination of AB to one of the principle axis at A is $\frac{1}{2}tan^{-1}\frac{3ab}{2(a^2-b^2)}$.

(iii) A particle is projected from the Earth's surface with velocity v. Show that if the diminution in gravity be taken into account and resistance of the air is neglected, the path is an ellipse of major axis $\frac{2ga^2}{2ga-v^2}$, where a is the Earth's radius and v is such that the orbit is an ellipse.

(iv) A square hole is punched out of a circular lamina, the diagonal of the square being a radius of the circle. Show that the centre of gravity of the remainder is at a distance $\frac{a}{8\pi-4}$ from the centre of the circle of diameter a.

***

ACTIVITY CODE: 1903126021

B.Sc. 6th Semester (Programme) Examination, October 2020

Subject: Mathematics

Course ID: 62118 **Course Code: SP/MTH/601/DSE-1B**

Course Title: Probability and Statistics

Full Marks: 20 **Time: 1 Hour**

<u>The figures in the margin indicate full marks</u>

Unless otherwise mentioned the symbols have their usual meaning.

1. **Answer *any two* of the following questions:** $(2 \times 2 = 4)$

a) Write down the axiomatic definition of probability.

b) If *A* and *B* are two independent events, then prove that A^C and *B* are also independent.

c) Show that the probability that exactly one of the events *A* and *B* occurs is $P(A) + P(B) - 2P(A \cap B)$.

d) Define the 'random variable' with an example.

e) For a random variable X, show that $Var(aX + b) = a^2 Var(X)$ for any $a, b \in \mathbb{R}$.

f) Write down the joint moment generating function and characteristic function of a 2D normal variable (X, Y).

g) State the central limit theorem.

h) Define un-biased estimate and consistent estimate.

2. **Answer *any two* of the following questions:** $(2 \times 5 = 10)$

a) Let (X, Y) be a bivariate random variable with joint distribution function

$$f(x,y) = \begin{cases} 6(1 - x - y), & \text{for } x > 0, \ y > 0, \ x + y = 1 \\ 0, & \text{otherwise} \end{cases}.$$

Then (i) show that X and Y are not independent.

(ii) Find $P\left(X + Y < \frac{1}{2}\right)$. (2+3)

b) (i) Let m be the mean of a random variable X and $a \in \mathbb{R}$. Show that

$$E\{(X-a)^2\} \geq E\{(X-m)^2\}.$$

(ii) Find the mean of binomial distribution. (2+3)

c) (i) Show that sample variance is a biased estimator of the population variance.

(ii) Find an un-biased estimator of population variance. (3+2)

d) If the probability density function $f(x)$ of a random variable X is defined by

$$f(x) = \frac{1}{2}e^{-|x|}, \quad -\infty < x < \infty,$$

Calculate the mean and variance of X. (5)

e) Find the value of the constant K such that

$$f(x) = \begin{cases} Kx(1-x), & 0 \le x \le 1 \\ 0, & elsewhere \end{cases}$$

is a possible probability density function and compute $P\left(X > \frac{1}{2}\right)$. (5)

f) Show that the function $f(x, y)$ defined by

$$f(x,y) = \begin{cases} \sin x \sin y, & 0 < x < \frac{\pi}{2}, 0 < y < \frac{\pi}{2} \\ 0, & elsewhere \end{cases}$$

is a possible two dimensional probability density function. Find the marginal density function and prove that the random variables are independent. (5)

3. Answer *any two* from either (a) or (b): **(**2×3=6)

(a) (i) Let X and Y be two random variables with correlational coefficient $\rho(X, Y)$. Show that

$$-1 \le \rho(X, Y) \le 1.$$

(ii) Let X and Y be two random variables with joint distribution function

$$f(x,y) = \begin{cases} x + y, & 0 < x < 1, \quad 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Then find $E(X, Y)$.

(iii) Show that the following function is a possible density function of a random variable:

$$f(x) = \begin{cases} |x|, & -1 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Also find the corresponding distribution function.

(iv**)** Find the mean and variance of Poisson distribution**.**

(b) (i) An un-biased die is thrown. Find the mathematical expectation of the number on the face appeared.

(ii) Calculate the correlation coefficient between X and Y from the given data

X	-1	0	1
Y	1	0	1

(iii) If T is an un-biased estimate of a population parameter θ, show that T^2 is a biased estimate of θ^2.

(iv) The regression equation of two random variables X and Y are $2x + 3y = 5, 5x + 2y = 8$. Find the correlation coefficient between X and Y.

***