SYLLABUS

For

M.A./M.Sc. in MATHEMATICS

Four Semesters

(Effective from the academic session 2019 – 2020 and onwards)

DEPARTMENT OF MATHEMATICS

BANKURA UNIVERSITY

BANKURA

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Syllabus of M.Sc. Mathematics

[With effect from 2019-20]

The duration of M.Sc. course of studies in Mathematics shall be of two years consisting of four semesters each of six months duration leading to Semester-I, Semester-II, Semester-III and Semester-IV examinations at the end of each semester. Syllabus for M.Sc. courses in Mathematics is hereby framed according to the following schemes and structures.

The scheme of course will be Choice Based Credit System (CBCS) as per UGC guidelines. Total marks (total credits) for M.Sc. course is 1050 (84) with 250 marks (20 credits) in each of the 1st, 2nd and 4th semesters comprising of five papers each of 50 marks (4 credits) and 300 marks (24 credits) in the 3rd semester comprising of six papers each of 50 marks (4 credits). In each theoretical paper 20% marks is allotted for Internal Assessment. The subject teacher/(s) will evaluate the internal assessment. All the students will have to take the compulsory core papers, four major elective papers, one minor elective paper and the term paper, which are distributed over four semesters.

The "Foundation Courses" will run in Semester-I & in Semester-II. Students will select any one of the given papers as "Foundation Course" in each of those semesters. The foundation courses are to be conducted by the other Departments of the University. The courses shall have internal assessment only, and so, credit earned from these courses shall not be considered while preparing the final result. However, the candidates are required to be successful to obtain "**Satisfactory"** (as against "**Not Satisfactory")** grade to become eligible for the fourth semester examination/award of the M.Sc. degree.

The major elective courses that will run in a particular year in Semester-III & Semester-IV will be decided by the Department. The students have to give options for taking two major electives in each of the last two semesters from the clusters of elective papers offered in those semesters. Students have to take same title (if arises) of major elective courses both in Semester-III and Semester-IV. The option norm for selection of major elective courses is to be framed by the Department in each year based on the SGPA/CGPA available from the previous semester(s) of the students. However, the distribution of students to the major elective courses will be equally divided as far as practicable. All faculty members will supervise the students for term paper. Students will be almost equally distributed among the supervisors for the term paper. Term paper will be done from any topic on Mathematics and its Applications. The marks distribution of the Term paper is 25 Marks for the dissertation, 15 Marks for Seminar presentation and 10 Marks for Viva-Voce. The supervisor and external expert together will evaluate the term paper.

Programme Objectives (PO):

PO1: Mathematical Knowledge: Apply the knowledge of mathematics to the solution of complex problems in academia and in real life.

PO2: Problem analysis: Identify, formulate, analyze/solve problems leading to conclusions using principles of mathematics.

PO3: Design/development of solutions: Design and develop methods and procedures for solutions of complex problems which meet the specified needs in industry, academia and real life.

PO4: Conduct investigations of complex problems: Use research-based knowledge and research methods including theory, experiment and computation; analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO5: Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern mathematical tools including prediction and modeling to complex mathematical activities with an understanding of the limitations.

PO6: Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

PO7: Communication: Communicate effectively on scientific activities with the scientific community and with the society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO8: Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of scientific & technological change.

Programme Specific Outcomes:

The Department of Mathematics offers exciting opportunities to talented students holding a Bachelor’s degree for acquiring a rigorous and modern education in mathematics and for pursuing master's degree in both pure and applied mathematics. As a part of the Programme, the student as to complete 84 credits of courses including a "Term Paper", whose major part is kind of academic research (and does not involve class room teaching), in a chosen area of pure or applied mathematics. This Program will introduce the students to modern as well as classical topics of mathematics, will help in acquiring thinking skills and to prepare him/her to undertake cutting-edge research in a Ph.D. Program.

Career Opportunities:

This program will enable the students to take part and qualify for the state and national level examinations such as SET, NET, GATE, NBHM etc. After completion of this program, the students are well prepared for higher education such as Ph.D. program and for a variety of jobs both in the industry and in academic institutions all over the world.

Course Structure

Sem. & Dur.	Course Type	Course Code	Name of the Course	Class hrs/week	L	T	P	Credit	I.A.	ESE
First Semester& 6 months	Core	Math-101C	Abstract Algebra	4	3	1	0	4	10	40
		Math-102C	Linear Algebra & Module Theory	4	3	1	0	4	10	40
		Math-103C	Real Analysis	4	3	1	0	4	10	40
		Math-104C	Ordinary differential equations and Partial differential equations	4	3	1	0	4	10	40
		Math-105C(IA)	Internal Assignment (Numerical Analysis and NA-Practical using C-Prog.)	6	1	1	4	4	10	40
	Comp. Found.	Math-106CF	Communicative Skill and personality development	1	1	0	0	1	50	
Second Semester& 6 months	Core	Math-201C	Complex Analysis	4	3	1	0	4	10	40
		Math-202C	Topology	4	3	1	0	4	10	40
		Math-203C	Calculus of several variables & Differential Geometry of curves and surfaces	4	3	1	0	4	10	40
		Math-204C	Techniques of Applied Mathematics (Generalized Functions, Special functions, Integral Equations)	4	3	1	0	4	10	40
		Math-205C(IA)	Internal Assignment (Integral Transforms & Computational methods for PDEs)	6	2	0	4	4	10	40
	Elect. Found.	Math-206EF	Yoga and Life Skills Education/ Value Education and Human Rights	1	1	0	0	1	50	

Sem. & Dur.	Course Type	Course Code	Name of the Course		Class hrs/week	L	T	P	Credit	I.A.	ESE
Third Semester& 6 months	Core	Math-301C	Functional Analysis		4	3	1	0	4	10	40
		Math-302C	Classical Mechanics		4	3	1	0	4	10	40
		Math-303C	Continuum Mechanics		4	3	1	0	4	10	40
	Major Elective -1 & Major Elective -2	Math-304ME & Math-305ME	Pure Group	Advanced Differential Geometry - I	4	3	1	0	4	10	40
				Operator Theory and Applications - I	4	3	1	0	4	10	40
				Advanced Algebra -I	4	3	1	0	4	10	40
			Applied Group	Basics of Mathematical Modelling	5	2	1	2	4	20	30
				Space science - I	4	3	1	0	4	10	40
				Dynamical Systems	4	3	1	0	4	10	40
	Minor Elective (Open Elective)	Math-306ME(ID)	Computer Applications		4	3	1	0	4	10	40

<table>
<tr><th>Sem. & Dur.</th><th>Course Type</th><th>Course Code</th><th colspan="2">Name of the Course</th><th>Class hrs/week</th><th>L</th><th>T</th><th>P</th><th>Credit</th><th>I.A.</th><th>ESE</th></tr>
<tr><td rowspan="10">Fourth Semester& 6 months</td><td rowspan="2">Core</td><td>Math-401C</td><td colspan="2">Operations Research</td><td>4</td><td>3</td><td>1</td><td>0</td><td>4</td><td>10</td><td>40</td></tr>
<tr><td>Math-402C</td><td colspan="2">Graph Theory & Field Theory</td><td>4</td><td>3</td><td>1</td><td>0</td><td>4</td><td>10</td><td>40</td></tr>
<tr><td colspan="2"></td><td colspan="2"></td><td colspan="7"></td></tr>
<tr><td rowspan="6">Major Elective-3
&
Major Elective-4</td><td rowspan="6">Math-403ME
&
Math-404ME</td><td rowspan="3">Pure Group</td><td>Advanced Differential Geometry – II</td><td>4</td><td>3</td><td>1</td><td>0</td><td>4</td><td>10</td><td>40</td></tr>
<tr><td>Operator Theory and Applications - II</td><td>4</td><td>3</td><td>1</td><td>0</td><td>4</td><td>10</td><td>40</td></tr>
<tr><td>Advanced Algebra -II</td><td>4</td><td>3</td><td>1</td><td>0</td><td>4</td><td>10</td><td>40</td></tr>
<tr><td rowspan="3">Applied Group</td><td>Space science - II</td><td>4</td><td>3</td><td>1</td><td>0</td><td>4</td><td>10</td><td>40</td></tr>
<tr><td>Modelling and Analysis of Biological systems</td><td>4</td><td>3</td><td>1</td><td>0</td><td>4</td><td>10</td><td>40</td></tr>
<tr><td>Computational Fluid Dynamics</td><td>4</td><td>3</td><td>1</td><td>0</td><td>4</td><td>10</td><td>40</td></tr>
<tr><td>Term Paper</td><td>Math-405T(IA)</td><td colspan="2">Internal Assignment (Mathematics and its Applications)</td><td>6</td><td>0</td><td>2</td><td>2</td><td>4</td><td>10</td><td>40</td></tr>
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Meaning of the symbols: ***L = Lecture*, *T = Tutorial*, *P = Practical*, *I.A. = Internal Assessment, ESE= End Semester Examination***

SEMESTER-I

Paper: Math-101C

Abstract Algebra

Course Objectives: The main objective of this course is to give a deep insight of group theory and ring theory. Converse of Lagrange's theorem will be discussed there in a great extent with some kind of special groups and their characterizations. This course also cover a good area of ring theory for their further use.

Course Specific Outcomes: After completion of this course a student would have

- a vast knowledge of group theory and ring theory which they can use for their further study,
- a clear idea of characterizations of groups and factorization domains.

Total Lectures: 50 ***(Marks – 50)***

Groups: Review of basic concepts of Group Theory, Homomorphism, Isomorphism and Automorphism of groups, Normal Subgroups, Quotient Groups, Isomorphism and Correspondence Theorems, Groups of order 4 and 6, D_4 and Q_8, Cayley's Theorem, Generalized Cayley's Theorem, Direct Product and Semi-Direct Product of Groups, Simple groups.

Group action on a set, Class equation, p-groups, Cauchy's theorem, Converse of Lagrange's theorem for finite Abelian groups, Sylow theorems and some of its applications, Normal and Subnormal Series, Composition Series, Solvable Groups and Nilpotent Groups, Jordan-Hölder Theorem and its applications, Finitely generated Abelian groups, free Abelian groups. (25L)

Rings: Ideals and homomorphisms, Prime and Maximal Ideals, Quotient Field of an Integral Domain, Polynomial Rings, Divisibility Theory: Euclidean Domain, Principal Ideal Domain, Unique Factorization Domain, Gauss' Theorem, Eisenstein's criterion. (25L)

References:

1. Dummit, Foote – *Abstract Algebra,* Wiley
2. Malik, Mordeson & Sen – *Fundamentals of Abstract Algebra,* McGraw-Hill
3. I. N. Herstein – *Topics in Algebra,* Wiley
4. P. B. Bhattacharya, S. K. Jain & S. R. Noyapal – *Basic Abstract Algebra,* Cambridge
5. Hungerford – *Algebra*, Springer

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Paper: Math-102C

Linear Algebra & Module Theory

Course Objectives: The main objective of this course is to study linear algebra and Module Theory. In linear algebra part, the crucial role of eigen values will be discussed here to characterize a linear operator. Also, various types of the canonical forms of matrices will be cultivated here. Basics of module theory will also be taught here.

Course Specific Outcomes: After completion of this course a student would have

- a clear knowledge of linear algebra regarding eigen values of various types of linear operators and their possible canonical forms along with linear functionals, bilinear forms and inner products,
- a basic idea of module theory.

Total Lectures: 50 ***(Marks – 50)***

Linear Algebra : Review of Vector Spaces, Euclidean spaces, Linear transformation in finite dimensional spaces and its matrix representation, rank and nullity, Linear functional, Dual space and Dual basis, Double Dual, Transpose of a linear transformation and matrix representation of the transpose of a linear transformation, Eigen values and Eigen vectors, Characteristic polynomials, Cayley-Hamilton theorem, Minimal polynomial, Invariant subspaces, Diagonalizability and Triangulability, Direct sum Decompositions, Projection of Linear transformations, Invariant Direct sums, Primary Decomposition Theorem.

Jordon Canonical Form, Rational Canonical Form, Computation of Invariant factors and Elementary divisors.

Inner Product spaces: Real and Complex Inner Product Spaces, Orthogonal vectors, Orthonormal Basis, Bessel's inequality, Gram-Schmidt Orthogonalization Process, Orthogonal Projection, Linear Functionals on an Inner Product Space and adjoints, Hermitian Operator, Unitary Operator, Normal Operator.

Bilinear Form and its Matrix Representation, Symmetric Bilinear Form, Quadratic Form, Classification of Quadric Forms, Sylvester's law of inertia. (40L)

Modules: Definitions of modules and examples, Module Homomorphisms, Submodules and Quotient Modules, Cyclic Modules, Free Modules, Artinian and Noetherian modules, Fundamental Structure Theorem (statement only) for finitely generated modules over a PID and its applications to decompositions of linear transformations over finite dimensional vector spaces. (10L)

References:

1. K. Hoffman & Kunze – *Linear Algebra* (Prentice – Hall)
2. Ramchandra Rao & P. Bhimasankar – *Linear Algebra* (TMH)
3. V. A. Iyin & Poznyak – *Linear Algebra* (Mir)
4. S. Lang – *Linear Algebra* (Springer Verlag)
5. Helmos – *Finite dimensional vector spaces*
6. Dummit, Foote – *Abstract Algebra,* Wiley
7. I. N. Herstein – *Topics in Algebra,* Wiley
8. Hungerford – *Algebra*, Springer

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Paper: Math-103C

Real Analysis

<u>Course Objectives:</u> The main objective of this course is to introduce measurable sets, measurable functions and study their various properties. The concept of Lebesgue integration as a generalization of Riemann integration will be introduced and cultivated here.

<u>Course Specific Outcomes:</u> After completion of this course a student would have

- a vast knowledge of measurable sets, measurable functions.
- a clear concept of Lebesgue integration,
- an idea how to construct non-measurable sets.

Total Lectures: 50 ***(Marks – 50)***

Functions of Bounded Variation and their properties, Riemann Stieltjes integrals and its properties.

Lebesgue outer measure, measurable sets and their properties, Borel sets, Cantor's Set, existence of non-measurable sets, Lebesgue measure, measurable functions and their properties, sequence of measurable functions, Egoroff's theorem, Applications of Lusin Theorem.

Simple and Step Functions, Lebesgue integral of simple and step functions, Lebesgue integral of a bounded function over a set of finite measure, Bounded Convergence Theorem, Lebesgue integral of non-negative function, Fatou's Lemma, Monotone Convergence Theorem. The General Lebesgue integral: Lebesgue Integral of an arbitrary Measurable Function, Lebesgue Integrable functions. Dominated Convergence Theorem.

Riemann Integral as Lebesgue Integral. (50L)

References:

1. I. P. Natanson – *Theory of Functions of a Real Variable,* Vol. I
2. C. Goffman – *Real Functions*
3. Burkil&Burkil – *Theory of Function of a Real Variable*
4. Goldberg – *Real Analysis*
5. Royden – Real Analysis
6. Limaye – *Functional Analysis*
7. Vulikh – *Function of a Real Variable*

8. Lahiri& Roy – *Theory of Functions of a Real Variable*
9. Apostol – *Real Analysis*
10. Shah &Saxena – *Functions of a Real Variable*
11. Charl'sScwarz – *Measure, Integration and Function space*
12. W. Rudin - *Real and complex analysis.*

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.
Internal Assessment – 10 marks.

Paper: Math-104C

Ordinary Differential Equations and Partial Differential Equations

Course Objectives: The aim of this course is to teach the students about Ordinary Differential Equations (ODEs) (both scalar and system of ODEs, linear and nonlinear ODEs), existence & uniqueness of solution, phase plane, critical points, stability of solutions. They also learn about Partial Differential Equations (PDEs), their degree & order, classification, existence and uniqueness of solution and different analytical methods for solving PDEs.

Course Specific Outcomes: After completion of this course, the students learn about:

- Linear and nonlinear (system of) ODEs, existence & uniqueness of solution with the help of Picard's and Piano's theorems,
- Solution techniques for both initial and boundary value problems for ODEs, stability of solutions,
- Strum-Liouville problem, Green's functions and orthogonal properties of solutions of Strum-Liouville problem,
- PDEs of first and second order, their classification, methods of solution of linear and nonlinear PDEs,
- Cauchy problem, method of characteristic, fundamental solution, Green's functions.

Total Lectures: 50 ***(Marks – 50)***

ODE: First order system of equations: Well-posed problems, simple illustrations. Peano's and Picard's theorems (statements only).(2L)

Linear systems, non-linear autonomous system, phase plane, critical points, stability, Liapunov stability, undamped pendulum.(5L)

Linear ordinary differential equations, generalized solution, fundamental solution, inverse of a differential operator.

Nonhomogeneous ordinary differential equations, Two-point boundary value problem for a second-order linear nonhomogeneous O.D.E. Self adjoint operator. Sturm-Liouville problem. Construction of Green's functions with examples. Orthogonal property of solutions of Sturm-Liouville problem.

Analogy between linear simultaneous algebraic equations and linear differential equation. (3L)

Partial Differential Equation: Partial Differential Equation of the first order. Cauchy's problem for the first order equations. Linear first order equations. Lagrange's equation. General solution. Integrals passing through a given curve. Orthogonal surfaces. Nonlinear PDE of the first order. Cauchy's method of Characteristics. Charpit's method. Examples.(7L)

Second order linear P.D.E: Classification, reduction to normal form; characteristic curves. Solution of linear hyperbolic equations by Riemann method. Riemann-Green's function. (7L)

Laplace equation: Occurrence of Laplace equations. Boundary value problems: Dirichlet (interior and exterior) and Neumann (interior and exterior). Separation of variables. Green's function for Laplace's equation. Its properties and methods of construction. (10L)

Wave equation: Occurrence of wave equation. D'Alemberts solution. Riemann-Volterra solution of one dimensional wave equation. Domain of influence and domain of dependence. Solution by separation of variables method. Solution by method of integral transforms. Vibrating membranes. (8L)

Diffusion equation: Occurrence of diffusion equation. Elementary solutions. Solution by integral transform technique. Separation of variables for rectangular and circular plate problems. Green's function. Bilinear expansion for Green's function. Finding the elementary solution of diffusion equation. (8L)

References:

1. I. N. Sneddon – *Integral Transforms* (MacGraw-Hill)
2. T.Amarnath – *Partial Differential Equation*
3. I. N. Sneddon – *Partial Differential Equation*
4. P. Phoolan Prasad & R. Ravichandan – *Partial Differential Equations*
5. F. John – *Partial Differential Equations*
6. Williams - *Partial Differential Equations*
7. Epstein - *Partial Differential Equations*
8. Chester - *Partial Differential Equations*.

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Paper: Math-105C(IA)

Numerical Analysis & Numerical Analysis Practical using C-Programming

Course Objectives: The aim of this course is to teach the students about different methods for finding numerical solution of different mathematical problems, convergence criteria of such solutions, analysis of the numerical methods, errors involved in the solution and to write computer program using C-Language for finding numerical solutions.

Course Specific Outcomes: After completion of this course, a student will be able to:

- Derive numerical methods for approximating the solution of different problems of mathematics,
- Analyze the error incumbent in any such numerical approximation,
- Implement a variety of numerical algorithms using appropriate technology
- Compare the viability of different approaches to the numerical solution of problems arising in solution of linear and nonlinear system of equations, eigen value problems, Ordinary differential Equations, etc.
- Write any source program to compute the numerical solutions of the mathematics problems, which arise in the research studies with applications to engineering, physical, biological or social sciences.

Group A

Numerical Analysis

Total Lectures: 20 ***(Marks – 20)***

Solving system of linear equations: Existence & uniqueness of solutions. *Iterative Methods*: Review of iterative methods, SOR methods. Operational count: operational counts of exact methods and iterative methods (3L), Multigrid Methods (3L).

System of Non-Linear Equations: Fixed point theory, Conditions of convergence, Newton's Method (4L), Variations of Newton's method. (3L)

Interpolation: Spline interpolation (3L)

ODE: IVP: Concept of singlestep and multistep methods; Review of solution methods for IVP, Generating solution curve of IVP using Runge-Kutta methods, Milne's method, Adam-Basforth method. (4L)

References:

1. F. B. Hildebrand – *Introduction to Numerical Analysis*

2. Demidovitch and Maron – *Computational Mathematics*
3. Jain, Iyengar and Jain – *Numerical Methods for Scientific and Engg. Computation*
4. A. Gupta and S. C. Basu – *Numerical Analysis*
5. Scarborough – *Numerical Analysis*
6. Atkinson – *Numerical Analysis*
7. Raulstan – *Numerical Analysis.*

Evaluation: End semester examination – 16 marks, 02 questions to be answered out of 03 questions carrying 08 marks of each.
Internal Assessment – 04 marks.

Group B

Numerical Analysis Practical using C-Programming

Total Lectures: 30 ***(Marks – 30)***

Practical: Using C programming
Addition, power, GCD, finding maximum among some numbers, finding prime numbers, generating Fibonacci Series, Matrix addition, Matrix multiplication. (5L)
Converting a Matrix into its row reduced echelon form/Lower or Upper triangular form if nonsingular square matrix, Calculation of Determinant of a Square matrix, Detection whether it is singular or not, Gauss Jordan Elimination. (7L)
Solving a system of equations with the coefficient matrix of the form of lower triangular matrix/ Upper triangular matrix; LU decomposition, Solution using LU decomposition, Gauss Elimination. (7L)
Gauss Jacobi iteration, Gauss-Siedel Iteration, Largest Eigen Value by Power method. (6L)
Using Runge-Kutta method for solving a differential equation with proper initial condition and plotting the solution. (5L)

References:

1. Xavier, C. – *C Language and Numerical Methods* (New Age International (P) Ltd. Pub.)
2. F. Scheid – *Computers and Programming* (Schaum's series)
3. *Gottfried, B. S. – Programming with C* (TMH)
4. Balaguruswamy, E. – *Programming in ANSI C* (TMH).

SEMESTER-II

Paper: Math-201C

Complex Analysis

Course Objectives: The main objective of this course is to give a deep insight of complex analysis. Complex integration will be the main focus this course. Various types of singularities will be introduced and cultivated here.

Course Specific Outcomes: After completion of this course a student would have

- a vast knowledge of various types of complex integrations,
- a clear idea of various types of singularities, their interconnections and their use in complex analysis.

Total Lectures: 50 ***(Marks – 50)***

Complex Plane, Stereographic Projection, Cauchy-Riemann equations, Analytic functions, Entire functions, Harmonic conjugates, Complex integration, line integral and its basic properties, index of a curve, winding number, connectedness of the complex plane, locally constant and globally constant functions, Cauchy's integral formula and higher derivatives, power series expansion of analytic functions. Zeros of analytic functions and their limit points, identity theorem, entire functions, Liouville's theorem, fundamental theorem of algebra, Maximum modulus principle, Schwarz's Lemma and its applications. Simply connected region and primitives of analytic functions, Morera's theorem, open mapping theorem. Singularities, Laurent's series expansion and classification of singularities and Casorati-Weierstrass's theorem, Cauchy's residue theorem and evaluation of improper integrals. Argument principle, Rouche's theorem and its application, maximum modulus theorem, Conformal maps, Mobius Transformations. (50L)

Text Book:

1. J. B. Conway – *Functions of one Complex Variable*(Narosa Publishing House)
2. Churchil, Brown – *Complex Variable* (MH)

References:

3. R. B. Ash – *Complex Variable* (A.P.)

4. Punoswamy – *Functions of Complex Variable*
5. Titchmarsh – *Theory of Functions.*

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Paper: Math-202C

Topology

<u>Course Objectives:</u> After the introduction of topological spaces, in this course, to study various properties and interconnections of different topological spaces is the main objective here. Connectedness, compactness and separation axioms will be studied here in details.

<u>Course Specific Outcomes:</u> After completion of this course a student

- would have a vast knowledge of various topological spaces which they can use for their further study,
- can see various structures studied before as special cases of topological spaces.

Total Lectures: 50 ***(Marks – 50)***

Topological spaces and Continuous Functions:

Topology on a set, Examples of Topologies (Topological Spaces): Discrete Topology, Indiscrete Topology, Finite Complement Topology, Countable Complement Topology, Topologies on the Real Line, Finer and Coarser Topologies, Basis and Sub basis for a topology, Product topology on $X \times Y$, Subspace Topology, Order Topology.

Interior Points, Limit Points, Derived Set, Boundary of a set, Closed Sets, Closure and Interior of a set, Kuratowski closure operator and the generated topology.

Continuous Functions, Rules for Constructing Continuous Functions: Inclusion Map, Composition, by restricting the domain, by restricting/expanding the range, Pasting Lemma, Open maps, Closed maps and Homeomorphisms.

(Infinite) Product Topology: Sub basis for product Topology defined by Projection Maps, Box Topology, Quotient Topology, Metric Topology.

Connectedness and Compactness:

Connected and Path Connected Spaces: Definitions, Examples and its properties, Connected subsets of the real line, Components and Path Components, Local Connectedness.

Compact space, Characterization of compact spaces in terms of finite intersection properties, Continuous image of compact spaces, Compact subsets of the real line, Heine-Borel Theorem, locally compact space.

Countability Axioms and Separation axioms: T_0, T_1, T_2 Spaces, Regular spaces, Completely regular spaces, Normal spaces, their properties and their relationships, Urysohn's lemma, Uryshon's Metrization Theorem, Tietze's extension theorem (Statement only) (50L).

Text Book:

1. J. R. Munkres – *Topology, a first course*

References:

2. W. J. Thron – *Topological Structures*
3. K. D. Joshi – Introduction to *General Topology*
4. J. L. Kelly – *General Topology*
5. G. F. Simmons – *Introduction to Topology & Modern Analysis.*

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Paper: Math-203C

Calculus of Several Variables & Differential Geometry of Curves and Surfaces

Course Objectives: The main objective of this course is to study geometry of curves and surfaces with the help of analysis of functions of several variables. This course gives a motivation to study advanced differential geometry.

Course Specific Outcomes: After completion of this course a student would have

- a clear idea of analytical properties of functions of several variables which they can use further,
- an idea of applications of geometry in advanced level.

Group A

Calculus of Several Variables

Total Lectures: 30 ***(Marks – 30)***

R^n as a normed linear space and $L(R^n, R^m)$ as a normed linear space, Limits and continuity of functions from R^n to R^m, The derivative at a point of a functions from R^n to R^m as a linear transformation, The tangent space and linear approximation. The chain rule.

Partial derivatives and higher order partial derivatives and their continuity, Sufficient conditions for differentiability, Comparison between the differentiability of a function from R^2 to R^2 and from C to C, Examples of discontinuous and non-differentiable functions whose partial derivatives exist, C^1 maps, Euler's theorem, Sufficient condition for equality of mixed partial derivatives, Proofs of the Inverse Function Theorem, the Implicit Function Theorem, and the Rank Theorem, Jacobians, The Hessian and the real quadratic form associated with it. Extrema of real-valued functions of several variables, the Lagrange multiplier condition for constrained extrema, Riemann Integral of real-valued functions on Euclidean spaces, measure zero sets, Fubini's Theorem, Partition of unity, change of variables, Stokes' Theorem and Divergence Theorem for integrals. (30L)

Text Book:

1. M. Spivak–*Calculus on manifolds*, Addison-Wesley Pub. Comp.
2. J. R. Munkres–*Analysis on manifolds*, Addison-Wesley Pub. Comp., 1991.

Reference Books:

1. Apostol, T.M., Mathematical Analysis, Narosa Publishing House, 2002.

2. Apostol, T.M., Calculus Vol I & II, John Wiley & sons, 2011.
3. R. Courant and F. John –*Introduction to calculus and analysis, Volume-II*, Springer-Verlag, New York, 2004.
4. Fleming, W., Functions of Several Variables, 2nd Edition, Springer-Verlag, 1977.
5. Kaplan, W., Advanced Calculus, Pearson, 2002.
6. Ghorpade, S.R. and Limaye, B.V., A Course in Multivariable Calculus and Analysis, Springer, 2009.

Group B

Differential Geometry of Curves and Surfaces

Total Lectures: 20 ***(Marks – 20)***

Tensors: Tensor and their transformation laws, Tensor algebra, Contraction, Quotient law, Reciprocal tensors, Kronecker delta, Symmetric and skew- symmetric tensors, Metric tensor, Riemannian space, Christoffel symbols and their transformation laws, Covariant differentiation of a tensor, Riemannian curvature tensor and its properties, Bianchi identities, Ricci-tensor, Scalar curvature, Einstein space.

Geometry of Curves: Definition of curves in R^n with examples, arc-length, reparametrization, level curves and parametric curves, curvature of plane curves and space curves, properties of plane curves, torsion of space curves, basic properties of plane and space curves, Serret-Frenet formulae.

Geometry of Surfaces: Definition of surfaces with various examples, smooth surfaces with examples, tangent, normal and orientability of surfaces, quadric surfaces. First fundamental form, length of curves on surfaces, surface area. Curvature of surfaces, second fundamental form. (20L)

Text Book:

1. Andrew Pressley– Elementary Differential Geometry, Springer-Verlag, 2001, London (Indian Reprint 2004).

Reference Books:

1. Manfredo P. Do Carmo– Differential Geometry of Curves and Surfaces, Prentice-Hall, Inc., Englewood, Cliffs, New Jersey, 1976.
2. Barrett O'Neill– Elementary Differential Geometry, 2^{nd} Ed., Academic Press Inc., 2006.

Evaluation: End semester examination – 40 marks.

Group A - 03 questions to be answered out of 05 questions carrying 08 marks of each.

Group B - 02 questions to be answered out of 03 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Paper: Math-204C

Techniques of Applied Mathematics

<u>Course Objectives:</u> The aim of this course is to teach the students about generalized functions, their elementary properties, mathematical operations, such as differentiation and regularization, on them. Students also learn about special functions in this course. The other important mathematical topic they learn is about integral Equations; their origin and classification, existence & uniqueness of solution of Volterra and Fredholm integral equations.

<u>Course Specific Outcomes:</u> After completion of this course, the students learn about:

- Generalized functions, their basic properties, relationship with the sequence of good functions, their differentiation,
- Special functions, ordinary point and singular point, Hermite polynomial, Legendre polynomial, Rodrigue's formula,
- Legendre functions, Bessel functions, solution of Legendre, Laugurre and Bessel equation,
- Origin and classification of integral equations, existence and uniqueness of Volterra, Fredholm & Abel's integral equations.

Total Lectures: 50 ***(Marks – 50)***

Generalized Functions: Distributions, Generalized functions and its elementary properties; Addition, Multiplication, Transformation of variables. Generalized function as the limit of a sequence of good functions, Differentiation of generalized function. Simple examples, Dirac-Delta function, Plemelz' formula.

Antiderivative, Regularization of divergent integral: Simple examples (8L)

Special functions: Ordinary point and singularity of a second order linear differential equation in the complex plane; solution about an ordinary point, solution of Hermite equation, Hermite polynomial; Regular singularity, Fuch's theorem, solution about regular singularity with examples, Frobenius' method, Solution of Hypergeometric equation.(6L)

Legendre polynomial, its generating function; Rodrigue's formula, recurrence relations and orthogonality properties; Associated Legendre functions, Legendre functions of second kind, expansion of a function in a series of Legendre Polynomials, spherical harmonics, Graph of Legendre function.(8L)

Bessel functions of first and second kind, its generating function, recurrence relations, Modified Bessel functions and their recurrence relations, orthonormality. Bessel series, Graph of Bessel function. (7L)

Solution of Legendre, Laugurre and Bessel equation. (4L)

Integral Equations: Origin and classification. Reduction of Initial value and boundary value problems to integral equations, Existence and Uniqueness of solutions of Fredholm and Volterra Integral equations; examples, Solution of Fredholm integral equation with degenerate kernel, symmetric kernel. Fredholm alternative, Numerical solution of Fredholm integral equations,Volterra integral equation of first and second kind. Resolvent kernel, Neumann series, Solution by Method of successive approximations, Difference kernel, Laplace Transform method, Examples, Numerical solution of Volterra integral equations, Singular integral equation, Solution of Abel's integral equation. (17L)

References:

1. Gelfand&Shilov – *Generalised Functions* (Academic Press)
2. E. D. Rainville – *Special Functions* (Macmillan)
3. I. N. Sneddon–*Special Functions of mathematical Physics & Chemistry* (Oliver & Boyd, London)
4. N. N. Lebedev–*Special Functions and Their Applications* (PH)
5. S. G. Mikhlin – *Integral Equation* (Pergamon Press)
6. F. G. Tricomi – *Integral Equation* (Interscience Publishers)
7. WE. V. Lovit. – *Linear Integral Equations* (Dover Publishers)

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Paper: Math-205C(IA)

Internal Assignment

Integral Transforms & Computational Methods for PDEs

Course Objectives: The aim of this course is to introduce Laplace, Fourier and Henkel transforms to the students, requirements on the functions for those methods to be applicable and the applications. Also, to teach the students about discretization techniques to find approximate solutions of differential equations, different types of errors involved in such solutions, their measures and practical applications.

Course Specific Outcomes: After completion of this course, students will:

- Students will gain a range of techniques employing the Laplace and Fourier Transforms in the solution of ordinary and partial differential equations. They will also have an appreciation of generalized functions, their calculus and applications.
- Students will learn discretization techniques for the solution of ordinary differential equations and partial differential equations; stability, consistency and convergence criteria of different discretization methods; practical application of the methods to some well-known PDEs arising from mathematical modelling of real-life problems.

Group – A

Integral Transforms

Total Lectures: 20 ***(Marks --20)***

Integral Transforms: Fourier Transform and its properties, Inversion formula of F.T.; Convolution Theorem; Parseval's relation. Finite Fourier transform and its inversion formula. Applications. (8L)

Laplace's Transform and its properties. Inversion by analytic method and by Bromwitch path. Lerch's Theorem. Convolution Theorem; Applications. Fourier and Laplace transform of generalized function.Applications, Hankel Transformations (12L)

Group – B

Computational Methods for PDEs

Total Lectures: 30 ***(Marks -- 30)***

Discretization Methods: Finite Difference Method for Partial Differential equations; Consistency, stability and convergence; Applications to Elliptic equations, Forward Difference Explicit Methods, Truncation errors for forward Difference Explicit Methods, Crank Nicolson Implicit method, Truncation errors for implicit methods; Applications to parabolic and hyperbolic equations.

References:

1. Analysis of Discretization Methods for Ordinary Differential Equations – Hans J. Stetter
2. Computational Fluid Dynamics: C. A. J. Fletcher (Springer)
3. Introduction to Computational Fluid Dynamics: P. Niyogi, S. K. Chakraborty, M. Laha.

Evaluation: End semester examination – 32 marks.

Group A - 02 questions to be answered out of 03 questions carrying 08 marks of each.

Group B - 02 questions to be answered out of 03 questions carrying 08 marks of each.

Internal Assignment – 08 marks.

Internal Assessment – 10 marks.

SEMESTER-III

Paper Math-301C

Functional Analysis

Course Objectives: The main objective of this course is study normed linear spaces, inner product spaces deeply. Banach spaces and Hilbert spaces and their various properties will be discussed here.

Course Specific Outcomes: After completion of this course a student would have

- a vast knowledge of Banach spaces and Hilbert spaces,
- an idea of Hahn-Banach theorem and its applications.

Total Lectures: 50 ***(Marks – 50)***

Banach Spaces: Normed Linear Spaces and its properties, Banach Spaces, Equivalent Norms, Finite dimensional normed linear spaces and local compactness, Riesz Lemma. Bounded Linear Transformations. Uniform Boundedness Theorem, Open Mapping Theorem, Closed Graph Theorem, Linear Functionals, Necessary and sufficient conditions for Bounded (Continuous) and Unbounded (Discontinuous) Linear functionals in terms of their kernel. Hyperplane, Necessary and sufficient conditions for a subspace to be hyperplane. Applications of Hahn-Banach Theorem, Dual Space, Examples of Reflexive Banach Spaces. Lp-Spaces and their properties.

Hilbert Spaces: Real Inner Product Spaces and its Complexification, Cauchy-Schwarz Inequality, Parallelogram law, Pythagorean Theorem, Bessel's Inequality, Gram-Schmidt Orthogonalization Process, Hilbert Spaces, Orthonormal Sets, Complete Orthonormal Sets and Parseval's Identity, Orthogonal Complement and Projections. Riesz Representation Theorem for Hilbert Spaces, Adjoint of an Operator on a Hilbert Space with examples, Reflexivity of Hilbert Spaces, Definitions and examples of Self-adjoint Operators, Positive Operators, Projection Operators, Normal Operators and Unitary Operators. Introduction to Spectral Properties of Bounded Linear Operators.

References:

1. Limaye, B.V., Functional Analysis, Wiley Eastern Ltd, 1981.

2. Kreyszig, E., Introductory Functional Analysis and its Applications, John Wiley and Sons, New York,1978.
3. Lusternik & Sobolov – *Elements of Functional Analysis*
4. B. K. Lahiri – *Elements of Functional Analysis*
5. Bachman and Narici – *Functional Analysis*
6. Brown & Page – *Functional Analysis*
7. W. Rudin – *Functional Analysis*

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.
Internal Assessment – 10 marks.

Paper: Math-302C

Classical Mechanics

Course Objectives: The aim of this course is to teach the students about Virtual work, D'Alembert's principal, work energy relations, Lagrangian formulation of Dynamics, Generalized coordinates, calculus of variation, configuration space and system point, generating functions, canonical transformations, Poission bracket, theory of small oscillations and special theory of relativity.

Course Specific Outcomes: After completion of this course, the students learn about:

- Constraints, basic problems with constraints, virtual work, D'Alembert's principal, work energy relations,
- Lagrangian formulation of dynamics, ignoration of coordinates, calculus of variation, invariance of Euler-Lagrange equation,
- Theory of small oscillations,
- Special Theory of Relativity, its postulates, Lorentz transformation, length contraction, time dilation, mass energy relation.

Total Lectures: 50 ***(Marks – 50)***

Constraints, Basic problems with constraint forces, Principle of Virtual Work, DAlembert's principle, Work energy relation for constraint forces of sliding friction.

Lagrangian formulation of Dynamics: Degrees of freedom, Generalized coordinates, Lagrange's equations of motion for holonomic and non-holonomic systems, Kinetic energy function, Theorem on total energy, linear generalized potentials, generalized momenta and energy. (5L)

Ignoration of co-ordinates, Routh's process for the ignoration of co-ordinates, Rayleigh's dissipation function. (6L)

Calculus of variation: Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema, Variational methods for boundary value problems in ordinary and partial differential equations, Brachistochrone problem. (6L)

Invariance of Euler-Lagrange equation of motion under generalized coordinate transformation. Configuration space and system point, Hamilton's principle, Hamilton's canonical equations of motion. Principle of energy, Principle of least action. (6L)

Generating Function, Canonical Transformations, Poisson Bracket. (4L)
Theory of small oscillations, Normal co-ordinates, Euler's dynamical equations of motion of a rigid body about a fixed point, Torque free motion, Motion of a top on a perfectly rough floor, Stability of top motion, Motion of a particle relative to rotating earth, Foucault's pendulum.(8L)

Special Theory of Relativity: Postulates of Special Relativity, Lorentz Transformation, Consequences of Lorentz Transformation, Lorentz group, Length contraction, time dilation, transformation of velocity, law of composition of velocity, mass variation, transformation of momentum, Energy mass relation$\boldsymbol{E = mc^2}$.(15L)

Reference:

1. F. Chorlton – *A Text Book of Dynamic*
2. Synge and Griffith – *Principles of Mechanics*
3. D. T. Green Wood – *Classical Dynamics*
4. E. T. Whittaker – *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies*
5. K. C. Gupta – *Classical Mechanics of Particles and Rigid Bodies*
6. I. S. Sokolnikoff – Mathematical Theory of Elasticity
7. T. J. Chung – *Continuum Mechanics* (Prentice – Hall)
8. Truesdell – *Continuum Mechanics* (Schaum Series)
9. Molla – *Theory of Elasticity*
10. F. Gantmacher – *Lectures in Analytical Mechanics*
11. J. L. Bansal – *Viscous Fluid Dynamics* (Oxford)
12. H. Goldstein – *Classical Mechanics*
13. R. N. Chatterjee – *Contunuum Mechanics*
14. N.C. Rana and P.S.Joag—Classical Mechanics

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.
Internal Assessment – 10 marks.

Paper: Math-303C

Continuum Mechanics

<u>Course Objectives:</u> The aim of this course is to teach the students about a unified treatment of all materials which can be treated as continua, the balance laws and the constitutive equations & the constraints that they should satisfy.

<u>Course Specific Outcomes:</u> After completion of this course, the students learn about:

- Lagrangian and Eulerian Coordinate systems
- Conservations laws
- Strain and stress tensors
- Constitutive equations for linear elastic materials
- Equations representing conservation principles
- Non-dimensionalization of the equations, non-dimensional parameters and their importance.

Total Lectures: 50 ***(Marks – 50)***

Continuous media: Deformation. Lagrangian and Eulerian coordinates. Relationship between Lagrangian and Eulerian Coordinates. Conservation of mass. Strain tensor. Rate of deformation tensor. Co-ordinate transformation of strains. Principal strains. Principal strain invariants. Examples. Maximum shear strains. Mohr circle representation. Compatibility equations.(12L)

Equilibrium and kinetics: Forces and stresses. Basic balance laws: Balance of linear and angular momentum; Cauchy's first and second laws of motion. Coordinate transformation of stresses. Principal stresses. Principal stress invariants. Examples. Mohr circle representation, the deviatoric stress tensor.(12L)

Constitutive equations for linear elastic solids. Generalized Hook's law. Monotropic, orthotropic, Transeversely isotropic and isotropic material. Lame constants. Navier equations. (6L)

Fluid flow problems: Definition of a fluid, Fluid properties, Classification of flow phenomena, Equations of fluid motion, Navier-Stokes equations (compressible and incompressible), Euler's equations (compressible and incompressible), Non-dimensionalization of equations, Reynolds number and Prandtl number. RANS equations. Introduction to turbulence and its modelling, introduction to DNS, LES and RANS simulations. (20L)

References:

1. T. J. Chung – *Applied Continuum Mechanics* (Prentice – Hall)
2. C. Truesdell – *Continuum Mechanics* (Schaum Series)
3. I. S. Sokolnikoff – *The Mathematical Theory of Elasticity (McGraw Hill)*
4. Molla – *Theory of Elasticity*
5. F. Gantmacher – *Lectures in Analytical Mechanics*
6. J. L. Bansal – *Viscous Fluid Dynamics* (Oxford)
7. H. Lamb – *A Treatise on the Mathematical Theory of the Motion of Fluids (Cambridge University Press)*
8. R. N. Chatterjee – *Contunuum Mechanics*
9. A. J. Chorin and J. E. Marsden – *Mathematical Introduction of Fluid Mechanics* (Springer)
10. G. K. Batchelor – *Fluid Dynamics* (Cambridge University Press)

Evaluation: End semester examination – 40 marks,

Continuous media: 03 questions to be answered out of 05 questions carrying 08 marks of each.

Fluid flow problems: 02 questions to be answered out of 03 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Papers: Math-304ME & Math-305ME

Advanced Differential Geometry-I

Course Objectives: This course is the first part of the two semester long elective course Advanced Differential Geometry. The main objective of this course is to study various types of manifolds with their topological connection. Lie groups and their properties will also be studied here.

Course Specific Outcomes: After completion of this course a student would have

- a vast knowledge regarding manifolds and their topological connection which will be used in the continuation of this course in the fourth semester,
- an idea of lie groups and their various properties.

Total Lectures: 50 ***(Marks- 50)***

Definition and various examples of differentiable manifolds, examples of non-Hausdorff, non-connected and non-2^{nd} countable manifolds, topology of manifolds, tangent spaces, cotangent spaces, Jacobian map, vector fields, integral curves, one parameter group of transformations, Lie derivatives, immersions and embeddings, distributions. (30L)

Multilinear maps, tensors, tensor products, tensor fields, exterior algebra, exterior derivatives. (10L)

Topological groups, Lie groups and Lie algebras, Lie subgroups, Heisenberg groups, product of two Lie groups, one parameter subgroups and exponential maps, examples of Lie groups, homomorphism and isomorphism, Lie transformation groups, general linear groups. (10L)

References:

1. B. B. Sinha, *An Introduction to Modern Differential Geometry*, Kalyani Publishers, New Delhi, 1982.
2. K. Yano and M. Kon, *Structure of Manifolds*, World Scientific Publishing Co. Pvt. Ltd., 1984.
3. John M. Lee, *Introduction to Smooth Manifolds*, 2^{nd} Ed., Springer-Verlag, 2012.

4. William H. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, Academic Press, New York, 1975.
5. S. Lang, *Introduction to Differential Manifolds*, John Wiley and Sons, New York, 1962.
6. S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry, Vol. 1,* Interscience Press, New York, 1969.

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.
Internal Assessment – 10 marks.

Operator Theory and Applications-I

Course Objectives: This course is the first part of the two semester long elective course Operator Theory and Applications. The main objective of this course is to study various types of operators, e.g., adjoint operators, compact operators on normed linear spaces and cultivate their properties in Banach and Hilbert spaces.

Course Specific Outcomes: After completion of this course a student would have

- a vast knowledge regarding adjoint, compacts, unitary, normal, positive operators in Hilbert spaces.

Total Lectures: 50 ***(Marks- 50)***

Adjoint operators over normed linear spaces, their algebraic properties, compact operators on normal linear spaces, sequence of compact operators, compact extensions, weakly compact-operators. (10L)

Operator equation involving compact operators, Fredhölm alternative, adjoint operators on Hilbert-spaces, self-adjoint operators and their algebraic properties; unitary operators, normal operators in Hilbert spaces, positive operators, their-sum, product; monotone sequence of positive operators, square-root of positive operator, projection operators. (15L)

Their sum and product, idempotent operators, positivity norms of projection operators; limit of monotone increasing sequence of projection operators. (25L)

References:

1. G. Bachman & L. Narici- *Functional Analysis*, Academic Press,1966
2. B.V. Limaye- *Functional Analysis*, Wiley Easten Ltd
3. E. Kreyszig-*Introductory Functional Analysis with Applications*, Wiley Eastern,1989
4. B.K. Lahiri-*Elements of Functional Analysis*, The world Press Pvt. Ltd., Kolkata, 1994
5. G.F. Simmons- *Introduction to topology and Modern Analysis* ,McGraw Hill, New York, 1963
6. N. Dunford and J.T. *Schwartz-Linear Operators, Vol-1&II*, Interscience, New York,1958
7. K. Yosida-*Functional Analysis*, Springer Verlog, New York, 3rdEdn., 1990
8. Brown and Page-*Elements of Functional Analysis*, Von Nostrand Reinhold Co., 1970
9. A.E. Taylor- *Functional Analysis*, John wiley and Sons, New York,1958
10. L.V. Kantorvich and G.P. akilov-*Functional Analysis*, Pergamon Press,1982

11. Vulikh- Functional Analysis
12. J. Tinsley Oden&Leszek F. Dernkowicz- *Functional Analysis*, CRC Press Inc, 1996.
13. Lipschitz-General Topology, Schaum Series.

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.
Internal Assessment – 10 marks.

Advanced Algebra - I

Course Objectives: This course is the first part of the two semester long elective course advanced studies of algebra. The main objective of this course is to study structure theorems of finitely generated modules over a PID and its applications to decompositions of linear transformations over finite dimensional vector spaces. Also local properties of commutative ring will be studied here with radicals.

Course Specific Outcomes: After completion of this course a student would have

- a vast knowledge regarding modules and its application on decompositions of linear transformations over finite dimensional vector spaces.
- a clear idea of commutative rings and their properties.

Total Lectures: 50 ***(Marks- 50)***

Module Theory:

Modules and Module homomorphisms, Submodules and Quotient Modules, Operations on submodules, Direct Sum and Product, Finitely Generated Modules, Free Modules, Artinian and Noetherian modules, Fundamental Structure Theorem for finitely generated modules over a PID and its applications to decompositions of linear transformations over finite dimensional vector spaces. Tensor Products of modules, Universal Property of the tensor product, Restriction and Extension of Scalars, Algebras. (20L)

Note: This course is based on Chapter 2 of [2] and Chapter 10 of [1].

Commutative Ring Theory:

Rings and Ring Homomorphisms, Ideals, Quotient Rings, Zero-divisors, Nilpotent elements, Units, Prime and Maximal ideals, Nilradical and Jacobson radical, Nakayama's Lemma, Operations on Ideals, Prime Avoidance, Chinese Remainder Theorem, Extension and Contraction of ideals. Rings and Modules of Fractions, Local Properties, Extended and contracted ideals in rings of fractions. Noetherian Rings, Primary Decomposition in Noetherian Rings. Integral Dependence, Lying-Over Theorem, Going-Up Theorem, Integrally Closed Domains, Going-Down Theorem, Noether Normalization, Hilbert Nullstellensatz. Transcendence Base, Separably Generated Extensions.

Note: This course is based on Chapters 1, 3, 4, 5 of [2]. (30L)

References:

1. Dummit, D.S., Foote, R.M., Abstract Algebra, Second Edition, John Wiley & Sons, Inc., 1999.

2. Atiyah, M., MacDonald, I.G., Introduction to Commutative Algebra, Addison-Wesley, 1969.
3. Lang, S., Algebra, Addison-Wesley, 1993.
4. Lam, T.Y., A First Course in Non-Commutative Rings, Springer Verlag.
5. Hungerford, T.W., Algebra, Springer.
6. Jacobson, N., Basic Algebra, II, Hindusthan Publishing Corporation, India.
7. Malik, D.S., Mordesen, J.M., Sen, M.K., Fundamentals of Abstract Algebra, The McGraw-Hill Companies, Inc.
8. Curtis, C.W., Reiner, I., Representation Theory of Finite Groups and Associated Algebras, Wiley-Interscience, NY.

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.
Internal Assessment – 10 marks.

Basics of Mathematical Modelling

Course Objectives: The main objective of this course is to generate the basic idea of mathematical modelling among the students. How to build a model, how to study it and how to test a model will be studied here.

Course Specific Outcomes: After completion of this course a student would have

- a clear idea how to formulate a mathematical model from mathematical and real life problem,
- a knowledge regarding formulation and study of a mathematical model.

Total Lectures: 50 ***(Marks- 50)***

Introduction: Description, objectives, classification and stages of mathematical modelling. (10L)

Model Building: Understanding the system to be modelled and its environment, system analysis, formulating the mathematical equations, solving equations efficiently. (10L)

Studying models: Dimensionless form, asymptotic behavior, sensitivity analysis, interpretation of model outputs. (10L)

Testing models: Verification of assumptions and model structures, prediction of unused data, estimating model parameters, comparing alternative models. (10L)

Using models: Predictions with estimates of precision, decision support. Discussion and practical applications. (10L)

References:

1. Michael D. Alder –*An Introduction to Mathematical Modelling* (HeavenForBooks.com)
2. AlfioQuarteroni–*Mathematical Models in Science and Engineering*, Notices of the AMS, Vol. 56(1), 2009.
3. Clive L. Dym– *Principles of Mathematical Modeling (2nd Edition)*, (Academic Press), 2004.
4. R. Illner, C. Sean Bohun, S. McCollum, T. van Roode–*Mathematical Modelling.*

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Dynamical Systems

Course Objectives: The main objective of this course is to study thoroughly dynamical systems. Linear system, non-linear system and its linearization, bifurcations, linear and non-linear difference equations will be taught in this course.

Course Specific Outcomes: After completion of this course a student would have

- a clear idea about linear and non-linear dynamical systems and how to linearize a non-linear system,
- a knowledge regarding bifurcations, linear and non-linear difference equations.

Total Lectures: 50 ***(Marks – 50)***

Linear systems:

Linear autonomous systems, existence, uniqueness and continuity of solutions, diagonalization of linear systems, fundamental theorem of linear systems, the phase paths of linear autonomous plane systems, complex eigen values, multiple eigen values, similarity of matrices and Jordon canonical form, stability theorem, reduction of higher order ODE systems to first order ODE systems, linear systems with periodic coefficients.

Nonlinear systems:

The flow defined by a differential equation, linearization of dynamical systems (two, three and higher dimension), Fixed Points, Stability: (i) asymptotic stability (Hartman's theorem), (ii) global stability (Liapunov's second method).

Periodic Solutions (Plane autonomous systems):

Translation property, limit set, attractors, periodic orbits, limit cycles and separatrix, Bendixon criterion, Dulac criterion, Poincare-Bendixon Theorem, index of a point, index at infinity.

Bifurcations:

Saddle-node, transcritical and pitchfork bifurcations, hopf- bifurcation.

Linear difference equations:

Difference equations, existence and uniqueness of solutions, linear difference equations with constant coefficients, systems of linear difference equations, qualitative behavior of solutions to linear difference equations.

Nonlinear difference equations (Map): Steady states and their stability, the logistic difference equation, systems of nonlinear difference equations, stability criteria for second order equations, stability criteria for higher order system. (50L)

References:

1. Beltrami, E., *Mathematics for Dynamic Modeling,* Academic Press, Orlando, Florida, 1987.

2. Kapur, J. N., *Insight into Mathematical Modeling,* Indian National Science Academy, New Delhi, 1983.

3. Meyer, W., *Concepts of Mathematical Modeling,* McGraw Hill, New York, 1994.

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Space Science--I

General Relativity and Cosmology

Course Objectives: The main objective of this course is to study thoroughly general relativity and generate a basic idea of cosmology.

Course Specific Outcomes: After completion of this course a student would have

- a clear idea about general relativity and cosmology,
- a knowledge of using mathematical tools in physics.

Total Lectures: 50 ***(Marks - 50)***

General Relativity: Minkowski space-time: Past and future Cauchy development, Cauchy surface. DeSitter and anti-de Sitter space-times. Robertson-Walker spaces. Spacially homogeneous space-time models. The Schwarzschild and Reissner – Nordstrom solutions. Kruskal diagram. Causal structure. Orientability. Causal curves. Causality conditions. Cauchy developments. Global hyperbolicity. The existence of Geodesics. The Causal boundary of space-time. Asymptotically simple spaces. (20L)

References:

1. *The large scale structure of space-time* - Hawking and Ellis (Camb. Univ. Press).
2. *General Relativit*y – R.M. Wald (Chicago Univ. Press).
3. *A first course in general relativity* – B.F. Schultz (Camb. Univ. Press).
4. *Gravitation and Cosmology* – S. Weinberg (J. Wiley and Sons).
5. *General Relativity, Astrophysics and Cosmology* – Raychaudhury, Banerji and Banerjee.(Springer-Verlag).
6. *General Relati*vity – M. Luduigsen (Camb. Univ. Press).
7. *Introducing Einstein's Relativity* – R d'Inverno (Clarendon Press, Oxford).

Cosmology: What is cosmology? Homogeneity and isotropy of the universe. The Weyl Postulate. The cosmological principle. General relativistic cosmological models. Cosmological observations. The Olbers Paradox. The Friedman Cosmological Models (dust and radiation models). Cosmologies with a non-zero λ. Hubble's Law, the age of the4 Universe. Gravitational red shift and Cosmological redshift. The spherically symmetric space-time: Schwarzschild solution. Patrick orbits in the Schwarzschild space-time. Newtonian approximation. Photon orbits. Birkhoffs theorem. Equilibrium of Massive spherical objects.

The Schwarzschild Interior solution. The interior structure of the star. Realistic stars and gravitational collapse. White dwarfs, Neutron stars. Gravitational collapse of a homogeneous dust ball. Schwarzschild black hole. Simple idea of black hole physics. (30L)

References:

1. *General Relativity and Cosmololgy* – J.V. Narlikar.
2. *A first course in general relativity* – B.F. Schultz.
3. *Introduction to cosmology* - J.V. Narlikar.
4. *An Introduction to Mathematical Cosmology* – J.N. Islam (Camb.Univ.Press).
5. *Gravitation and Cosmology* – S. Weinberg (J. Wiley and Sons.)
6. *General Relativity, Astrophysics and Cosmology* – Raychaudhuri, Banerji and Banerjee (Springer-Verlag).
7. *Introduction to Cosmology* – M. Ross (J. Wiley and Sons).

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Paper: Math-306ME(ID)

Computer Applications

Course Objectives: The aim of this course is to teach the students about theory of C-language and Matlab.

Course Specific Outcomes: After completion of this course, the students learn about:

- C-language and Matlab.

Total Lectures: 50 ***(Marks–50)***

Unit 1: Programming Language in C: C Character set, Keywords, Basic data types, Constants, Variables, Operators and Expressions, Assignment Statements, I/O statements. Control Statements: Decision making and Looping statements in C, break and continue statements, Example of Simple Programs.

Unit 2: Subscripted variables: Concept of array variables in programming language, Rules for one- dimensional and two-dimensional subscripted variable in C, String, Example of Simple programs.

Definition of Function, Built-in function, user-defined function, Recursion.

Unit 3: Introduction to MATLAB. (50L)

References:

1. *Programming in ANSI C* – E. Balagurusami, Tata McGraw-Hill.
2. *Let us C* – Y. Kanetkar, BPB Publication.
3. *Getting started with MATLAB 7*- Rudra Pratap, Oxford University Press.

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

SEMESTER-IV

Paper: Math401C

Operations Research

<u>Course Objectives:</u> The aim of this course is to teach the students about Operations Research (OR), its definition, applications in different sectors, formulation and solution of LPP, primal and dual simplex methods, formulation and solution of assignment problems, Basics of integer programming, project scheduling by PERT & CPM techniques, deterministic inventory problems.

<u>Course Specific Outcomes:</u> After completion of this course, the students learn about:

- Definition of OR, scope of OR, applications of OR,
- Formulation of LPP, solution using primal and dual Simplex methods,
- Mathematical formulation of Assignment problems, optimality conditions, different methods of solution,
- Project scheduling by PERT and CPM,
- Deterministic inventory control models, classification of inventories, features of inventory systems.

Total Lectures: 50 ***(Marks–50)***

Introduction, Definition of O.R., Drawbacks in definition, Scope of O.R., O.R. and decision making, Application of O.R. in different sectors, Computer application in O.R. Fundamental theorem of L.P.P. along with the geometry in n-dimensional Euclidean space, hyperplane, separating and supporting plane.

Revised simplex method with and without artificial variable, modified dual simplex, decomposition principal of Dantzig and Wolf, Sensitivity analysis, Bounded variable method.

Search methods: Fibonacci and Golden section methods. Gradient method: Method of conjugate directions for quadratic functions, Steepest descent and Davodon-Flecher-Powell method.

Integer Programming: Gomory's cutting plane algorithm, Gomory's mixed integer method, Branch and Bound method. Goal programming: Concept of Goal programming, Its graphical solution, Modified simplex method of Goal programming.

Network: Project scheduling by PERT/CPM, Introduction, Basic differences between PERT and CPM, Steps of PERT/CPM Techniques, PERT/CPM network Components and Precedence Relationships, Critical Path analysis, Probability in PERT analysis, Project Crashing, Time cost Trade-off procedure, Updating of the Project, Resource Allocation.
Deterministic Inventory control Models: Introduction, Classification of Inventories, Advantage of Carrying Inventory, Features of Inventory System, Deterministic inventory models with and without shortages. (50L)

References:

1. Wagner – *Principles of Operations Research* (PH)
2. Sasievir, Yaspan, Friedman – *Operations Research*: *Methods and Problems* (JW)
3. J. K. Sharma – *Operations Research – Theory and Applications*
4. Taha – *Operations Research*
5. Schaum's Outline Series – *Operations Research*
6. Hillie& Lieberman – *Introduction to Operations Research*
7. Swarup, Gupta & Manmohan –*Operations Research.*

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.
Internal Assessment – 10 marks.

Paper: Math402C

Graph Theory & Field Theory

Course Objectives: The main objective of this course is to study graph theory with various types of graphs and their application to mathematical and real world. Also in this course field extension along with Galois fields will be studied with their application to formulate a process of solving polynomial of any degree.

Course Specific Outcomes: After completion of this course a student would have

- a vast knowledge of graph theory and its applications to mathematical and real world,
- an idea of Galois field extension to formulate a process of solving polynomial of any degree.

Group A

Graph Theory

Total Lectures: 20 ***(Marks – 20)***

Graphs, subgraphs and their various properties and characterization, complement, isomorphism, walks, paths, cycles, connected graph components, bipartite graph.

Adjacency matrix, incidence matrix.

Directed graph, adjacency matrix of a digraph.
Distance, radius and center, diameter.

Degree sequence, Havel-Hakimi theorem (statement only).

Trees, various characterizations of trees, centres of trees, spanning trees, Fundamental cycles with respect to spanning tree, Cayley's theorem on trees.
Eulerian graphs, Hamiltonian graphs, Königsberg Bridge problem.
Coloring of graphs, Chromatic number, Chromatic polynomial, König theorem, Recurrence formulae.

Planar graphs, statement of Kuratowski Theorem, Eulers formula, 5 colour theorem, statement of 4 colour theorem, dual of aplanar graph.(20L)

References:

1. F. Harary – *Graph Theory* (Addison-Wesley, 1969)

2. D. West – *Introduction to Graph Theory*, PHI
3. J. A. Bondy U.S.R. Murty – *Graph Theory with Applications* (Macmillan, 1976)
4. Nar Sing Deo – *Graph Theory* (Prentice-Hall, 1974)
5. Malik, Sen, Ghosh - *Introduction to graph theory* (Cengage)
6. K. R. Pathasarthy – *Basic Graph Theory* (TMH., 1994).

Group B

Field Theory

Total Lectures:* 30** ***(Marks – 30)

Field Theory: Extension of fields, simple extensions, algebraic and transcendental extensions, splitting fields, algebraically closed fields, normal extension, separable extensions, Impossibility of some constructions by straightedge and compass, Galois field, perfect field, Galois group of automorphisms and Galois theory, solution of polynomial equations by radicals. (30L)

References:

1. Hungerford – *Algebra*, Springer
2. Patrick Morandi – *Field and Galois Theory,* Springer
3. Dummit, Foote – *Abstract Algebra,* Wiley
4. Malik, Mordeson&Sen – *Fundamentals of Abstract Algebra,* McGraw-Hill
5. I. N. Herstein – *Topics in Algebra,*Wiley
6. David A. Cox – *Galois Theory,* Wiley.

Evaluation: End semester examination – 40 marks.

Group A - 02 questions to be answered out of 03 questions carrying 08 marks of each.

Group B - 03 questions to be answered out of 05 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Paper: Math 403ME&Math 404ME

Advanced Differential Geometry-II

Course Objectives: This course is the continuation of the course Advanced Differential Geometry –I offered in third semester as a major elective paper. The main objective of this course is to study various advanced properties of Riemannian manifolds.

Course Specific Outcomes: After completion of this course a student would have

- a vast knowledge regarding affine and Riemannian connections,
- an idea of Riemannian manifolds and their various properties.

Total Lectures: 50 ***(Marks- 50)***

Affine connections, its existence, curvature, torsion of an affine connection, Riemannian manifolds, Riemannian connection, curvature tensors, sectional curvature, Schur's theorem, geodesics in a Riemannian manifold, projective curvature tensor, conformal curvature tensor. (30L)

Submanifolds & hypersurfaces,normals, Gauss' formulae, Weingarten equations, lines of curvature, generalized Gauss and Mainardi-Codazzi equations. (10L)

Almost complex manifolds, Nijenhuis tensor, contravariant and covariant almost analysis vector fields, F-connection. (10L)

References:

1. R. S. Mishra, *Structures on a differentiable manifold and their applications*, Chandrama Prakashan, Allahabad, 1984.
2. B. B. Sinha, *An Introduction to Modern Differential Geometry*, Kalyani Publishers, New Delhi, 1982.
3. K. Yano and M. Kon, *Structure of Manifolds*, World Scientific Publishing Co. Pvt. Ltd., 1984.
4. John M. Lee, *Introduction to Smooth Manifolds*, 2^{nd} Ed., Springer-Verlag, 2012.

5. William H. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, Academic Press, New York, 1975.
6. S. Lang, *Introduction to Differential Manifolds*, John Wiley and Sons, New York, 1962.
7. S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry*, Vol. 1, Interscience Press, Newyork, 1969.

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Operator Theory and Applications-II

<u>Course Objectives:</u> This course is the continuation of the course Operator Theory and Applications –I offered in third semester as a major elective. The main objective of this course is to study spectral properties of self- adjoint operators and compact normal operators Hilbert spaces.

<u>Course Specific Outcomes:</u> After completion of this course a student would have

- a vast knowledge regarding spectral theories of self- adjoint operators and compact normal operators Hilbert spaces.
- Also an idea regarding unbounded operators and their properties in Hilbert spaces.

Total Lectures: 50 ***(Marks- 50)***

Spectral properties of bounded-linear operators in normed linear space, spectrum, regular value, resolvant of operator, closure property and boundedness property of spectrum, spectral radius. (15L)

Eigenvalues, eigen-vectors of self-adjoint operators in Hilbert space, resolvant sets, real property of spectrum of self-adjoint operators, range of spectrum, orthogonal direct sum of Hilbert space. (15L)

Spectral theorem for compact normal operators, sesquilinear functionals, property of boundedness and symmetry, generalisation of Cauchy-Schwarz inequality. (10L)

Unbounded operators and their adjoint in Hilbert spaces. (10L)

References:

1. G. Bachman & L. Narici- *Functional Analysis*, Academic Press,1966
2. B.V. Limaye- *Functional Analysis*, Wiley Easten Ltd
3. E. Kreyszig-*Introductory Functional Analysis with Applications*, Wiley Eastern,1989
4. B.K. Lahiri-*Elements of Functional Analysis*, The world Press Pvt. Ltd., Kolkata, 1994
5. G.F. Simmons- *Introduction to topology and Modern Analysis* ,McGraw Hill, New York, 1963
6. N. Dunford and J.T. Schwartz-*Linear Operators, Vol-1&II*, Interscience, New York,1958
7. K. Yosida-*Functional Analysis*, Springer Verlog, New York, 3rdEdn., 1990
8. Brown and Page-*Elements of Functional Analysis*, Von Nostrand Reinhold Co., 1970

9. A.E. Taylor- *Functional Analysis*, John wiley and Sons, New York,1958
10. L.V. Kantorvich and G.P. akilov-*Functional Analysis*, Pergamon Press,1982
11. Vulikh- Functional Analysis
12. J. Tinsley Oden&Leszek F. Dernkowicz- *Functional Analysis*, CRC Press Inc, 1996.

Lipschitz-General Topology, Schaum Series.

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Advanced Algebra - II

Course Objectives: This course is the continuation of the course Advanced Algebra –I offered in third semester as a major elective. The main objective of this course is to study tensor properties of modules, exact sequences, injective, projective and flat modules. Also non-commutative ring will be studied here with Wedderburn - Artin theorem**.**

Course Specific Outcomes: After completion of this course a student would have

- a vast knowledge regarding tensor properties of modules, exact sequences, injective, projective and flat modules.
- a clear idea of non-commutative rings and their properties.

Total Lectures: 50 ***(Marks- 50)***

Module Theory: Tensor Products of modules, Universal Property of the tensor product, Restriction and Extension of Scalars, Algebras. Exact Sequences, Projective, Injective and Flat Modules, Five Lemma, Projective Modules and $\boldsymbol{Hom_R(M,\_)}$, injective modules and $\boldsymbol{Hom_R(\_\,,M)}$, Flat modules and $\boldsymbol{M \otimes_R \_}$. (20L)

Note: This course is based on Chapter 2 of [2] and Chapter 10 of [1].

Structure of Rings:

Artinian rings, Simple rings, Primitive rings, Jacobson density theorem, Wedderburn - Artin theorem on simple (left)Artinian rings. The Jacobson radical, Jacobson semisimple rings, subdirect product of rings, Jacobson semisimple rings as subdirect products of primitive rings, Wedderburn - Artin theorem on Jacobson semisimple (left)Artinian rings. Simple and Semisimple modules, Semisimple rings, Equivalence of semisimple rings with Jacobson semisimple (left)Artinian rings, Properties of semisimple rings, Characterizations of semisimple rings in terms of modules. (30L)

Note: This course is based on the books [4] and [3] Chapter XVII and chapter IX of [5].

References:

1. Dummit, D.S., Foote, R.M., Abstract Algebra, Second Edition, John Wiley & Sons, Inc., 1999.
2. Atiyah, M., MacDonald, I.G., Introduction to Commutative Algebra, Addison-Wesley, 1969.
3. Lang, S., Algebra, Addison-Wesley, 1993.
4. Lam, T.Y., A First Course in Non-Commutative Rings, Springer Verlag.

5. Hungerford, T.W., Algebra, Springer.
6. Jacobson, N., Basic Algebra, II, Hindusthan Publishing Corporation, India.
7. Malik, D.S., Mordesen, J.M., Sen, M.K., Fundamentals of Abstract Algebra, The McGraw-Hill Companies, Inc.
8. Curtis, C.W., Reiner, I., Representation Theory of Finite Groups and Associated Algebras, Wiley-Interscience, NY.

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.
Internal Assessment – 10 marks.

Computational Fluid Dynamics

Course Objectives: The aim of this course is to teach the students about hyperbolic conservation laws, the concept of weak solutions, entropy conditions, discretization methods for obtaining weak solutions, conservative, consistent and stable numerical schemes for computational solutions, large-scale computations of viscous incompressible flows.

Course Specific Outcomes: After completion of this course, the students learn about

- Cauchy problem for hyperbolic conservation laws
- Concept of weak, entropy solutions
- Concept of conservative, consistent and stable schemes for hyperbolic conservation laws
- Discretization methodologies that go into complex large scale fluid flow simulations.

Total Lectures: 50 ***(Marks – 50)***

Introduction/motivation: What is CFD, need of CFD; Basic equations of fluid dynamics: Compressible Navier-Stokes equations, compressible Euler equations, boundary conditions; Incompressible Navier-Stokes equations, incompressible Euler equations, Stokes equations, boundary conditions; Preliminary computational techniques: Concepts of finite difference, finite volume and finite element methods, similarities & distinctions of these three methods. (5L)

Practical applications (implementations) to parabolic and elliptic equations. (5L)

Hyperbolic conservation laws: Concept of weak solution, Rankine-Hugoniot condition, Entropy condition, Entropy solution. (4L)

Discretization schemes for scalar conservation laws: Naive-scheme and Lax-Friedrichsscheme, Schemes in conservation form, consistency, stability, equivalent equation, CFLcondition, Lax-equivalence theorem. (10L)

Second order schemes, Non-smooth solutions, Lax-Wendroff theorem, Upwind and Godunov schemes. (6L)

Computation of incompressible viscous flows: Unsteady flows, staggered grid, MAC method, Higher order upwind differencing. (10L)

Steady flows: Non-linear iterations, SIMPLE formulations, implementations (10L)

References:

1. Randall J. LeVeque – Numerical Methods for Conservation Laws (Birkhäuser)
2. Kröner, D. – Numerical schemes for conservation laws (Wiley-Teubner)
3. Fletcher C. A. J. – Computational Techniques for Fluid Dynamics (Springer)
4. C. Hirsch – Numerical Computation of Internal and External Flows: The Fundamentals of Computational Fluid Dynamics (Butterworth-Heinemann)
5. P. Niyogi, S. K. Chakraborty, M. Laha – Introduction to Computational Fluid Dynamics (Pearson Education)

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.
Internal Assessment – 10 marks.

Space Science –II

Astrophysics

Course Objectives: The main objective of this advanced course is to study astrophysics as an application of general relativity which was taught in its earlier course Space Science -I.

Course Specific Outcomes: After completion of this course a student would have

- a clear idea about astrophysics,
- a knowledge of Big bang theory.
- a knowledge of using mathematical tools in physics.

Total Lectures: 50 ***(Marks – 50)***

Application of General Relativity to Astrophysics.

Compact Objects, White dwarfs, Neutron stars and Black holes. Brief history of the formation and evolution of stars.

Schwarzschild exterior solution, Birkhoff's theorem, Schwarzschild singularity, Kruskal transformation, Schwarzschild Black hole. Motion of test particles around Schwarzschild black hole. Kerr metric and Kerr black holes (without deduction of solution). Horizons of Schwarzschild and Kerr black holes. Laws of black hole thermodynamics (statements only).

Interior of Schwarzschild metric, massive objects, Openheimer – Volkoff limit, Gravitational lensing, Quasars, Pulsars, Supernova.

Openheimer-Snydder non static dust model, Gravitational collapse.

Accretion into compact objects, Boltzmann formula, Saha Ionization equation, H-R diagram.

Plasma, black Body, Cherenkov &Synchroton Radiation.Accreation as source of radiation. Quasar as source of radition, Compton effecteffect, Bremsstrallung Radition.

Formation of Galactic Structure – different theories : Formation of our Galaxy. Formation of Galaxy in Evolutionary Universe. Formation of Galaxy in Steady State Universe. Possibility of galactic structure formation through Explosion.

Hubble's Law & Expansion of Universe – Big Bang Model. Uniformity of Large Scale Structure of the Universe. Origin of Cosmic Rays. Origin of Galaxies and the Universe.

References:

1. *The Structure of the Universe* – J.V. narlikar

2. *Astrophysics* – B. Basu

3. *Astrophysics Books*: ThanuPadmanabhan

4. *Astrophysical Concept* – M. Harmitt

5. *Galactic Structure* – A. Blauaw& M. Schmidtw

6. *Large Scale Structure of Galaxies* – W.B. Burton

7. *The Milky Way* – B.T. Bok & P.F. Bok.

8. *Cosmic Electrodynamics* – J.H. Piddington

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.

Internal Assessment – 10 marks.

Modeling and Analysis of Biological Systems

<u>Course Objectives:</u> The main objective of this course is to use mathematical modelling in various types of biological systems. In this bio-math course different types of discrete and continuous biological systems will be analyzed through different types of mathematical models.

<u>Course Specific Outcomes:</u> After completion of this course a student would have

- a clear idea how to use mathematical modelling in different types of biological systems,
- a knowledge of analyzing different types of mathematical models of biological systems.

Total Lectures: 50 ***Marks: 50***

The nature of ecosystems, Autotroph-based ecosystem, Detritus-based ecosystem, Different types of population growth, Community dynamics- succession and community responses.

Single Species Population Dynamics:

Continuous growth models – their stability analysis, Influence of random perturbations on population stability. Insect out break model- Spruce-Budworm model. General autonomous models. Delay Models.

Population Dynamics of Two Interacting Species:

Introduction, Lotka-Volterra system of predator-prey interaction, Trophic function, Gauss's Model, Gause Model, Kolmogorov Model, Leslie Gower Model, Analysis of predator-prey model with limit cycle periodic behavior, parameter domains of stability. Competition models- exclusion principle and stability analysis. Models on mutualism.

Continuous models for three or more interacting species:

Three species simple and general food chain models- its stability and persistence. Models on one prey two competing predators with limited resources and living resource supporting three competing predators- stability analysis and persistence.

Reaction - Diffusion equation, Turing stability, Pattern Formation.

References:

1. Kapur, J. N., *Insight into Mathematical Modeling,* Indian National Science Academy, New Delhi, 1983.
2. Perko, L., *Differential Equations and Dynamical Systems*, Springer Verlag, 1991.
3. Kelley, W. G., Peterson, A. C., *Difference Equations- An Introduction with*

Applications, Academic Press, 1991.

Evaluation: End semester examination – 40 marks, 05 questions to be answered out of 08 questions carrying 08 marks of each.
Internal Assessment – 10 marks.

Paper: Math-405T(IA)

<u>Course Objectives:</u> The aim of this course is to engage the student in a study or research on a topic of mathematics which is beyond the regular mathematics courses offered in regular class-room teaching in our Department. The students need to produce a document (paper or report) containing the result of this study and present the content orally to a group consisting of the faculty members and an external expert.

Course Specific Outcomes: After completion of this course, the students learn about:

- basic components of academic research such a literature survey, self study to identify a problem, solve it and produce report on his/her work etc.,
- Prepare a scientific presentation and deliver it to a group of audience consisting of faculty members,
- Prepare and successfully taking part in a viva voce.

Marks: 50

Internal Assignment (Term Paper) Math-405T(IA) is related with any topic of Mathematics and Applications and the Marks distribution is 25 Marks for written submission and 15 Marks for Seminar Presentation and 10 Marks for Viva-Voce.