

**B.Sc. 1<sup>st</sup> Semester (Honours) Examinations, 2020-21**

**PHYSICS**

**Course ID:12411**

**Course Code: SH/PHS/101/C-1**

**Course Title: Mathematical physics - I**

**Time: 1hour 15 minutes**

**Full Marks: 25**

*The figures in the margin indicate full marks*

*Candidates are required to give their answers in their own words as far as practicable*

***Section - I***

**1. Answer any five of the following questions: (1×5 = 5)**

- a) Evaluate  $\Gamma(-\frac{3}{2})$ .
- b) Write down the wronskian of a second order differential equation.
- c) Write down the Dirichlet conditions related to Fourier series.
- d) A force given by  $\vec{F} = 3\hat{i} + 2\hat{j} - 3\hat{k}$  is applied at the point (1,-2,3). Find the moment of  $\vec{F}$  about the point (-2,1,4).
- e) Show that  $\vec{E} = \frac{\vec{r}}{r^2}$  is irrotational.
- f) Write down two dimensional Laplace's equation in cylindrical co-ordinates( $r,\theta$ )
- g) Show that  $J_{-n}(x) = (-1)^n J_n(x)$ ,  $J_n(x)$  implies usual meaning.
- h) For the Error function (Probability integral) prove that  $erf(\infty) = 1$ .

*Please Turn Over*

## Section - II

**Answer any two of the following questions:**

**(5 × 2 = 10)**

2. Represent the vector  $\vec{A} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$  in cylindrical coordinate system and determine  $A_\rho, A_\phi, A_z$ ; symbols have their usual meaning. (2+3=5)

3. a) Prove that  $\vec{A} = r^2\vec{r}$  is conservative and find the scalar potential .

b) Evaluate  $\int_C \vec{A} \cdot d\vec{r}$  along the curve  $x^2 + y^2 = 1, z = 1$  in the positive direction from (0,1,1) to (1,0,1) if  $\vec{A} = (yz + 2x)\hat{i} + xz\hat{j} + (xy + 2z)\hat{k}$  (1+1 +3=5)

4. Solve the differential equation .  $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$  5

5. Find the Fourier series expansion of the function  $f(x) = x + x^2$  for  $-\pi < x < \pi$  .

From this expansion, show that,  $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$  (4+1=5)

## Section - III

**Answer any one of the following questions:**

**(10×1 =10)**

6.a) State Stokes theorem in vector calculus.

b) Verify the divergence theorem for  $\vec{A} = 2x^2y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$  taken over the region in the first octant bounded by  $y^2 + z^2 = 9$  and  $x = 2$ . (2+8=10)

7. a) Show that the Legendre polynomials can be represented by Rodrigue's formula:

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

b) Evaluate  $\int_0^1 \frac{dx}{\sqrt{1-x^n}}$

ii) Find out the relation between Beta and Gamma function. (4+2+4=10)

